modified by a prefix or suffix which is in the nature of a classifier to denote the class of objects counted. The Haida has no less than 15 of these classifiers. Others seem to have an even larger number.[1] This usage is found very generally among the languages of the north Pacific coast and but rarely in other parts of the country. The Tsimsian mentioned above has different forms for abstract counting, for counting flat objects or animals, for round objects or time, for men, for long objects, for canoes, for measures.

3. **Verbal Nature.** In a few languages the numerals are true verbs instead of adjectives, and as such are conjugated through all the variations of mood, tense, person and number. As far as found this peculiarity is limited to three languages, Cree, Crow, Micmac.

4. **Derivative Numerals.** The formation of ordinals, adverbials and distributives from the cardinal numerals is more properly a grammatical than a numerical process and as such need receive only slight notice here. (*a*) *Ordinals* are found most frequently, usually being formed from the cardinals by a suffix or other terminal modification, occasionally by a prefix. In the Creek an unusual method is found, the ordinal being formed from the cardinal in the same way that the superlative of the adjective is formed from the comparative. (*b*) *Distributives* are also frequently found. They are formed from the cardinals by prefixes, suffixes, or reduplication. (*c*) *Adverbials,* as far as noted, are formed by suffixes.

5. **Arithmetical Operations.** We shall close our discussion of the varied, interesting, and intricate number systems of the North American Indians with a reference to their ability to perform arithmetical operations. In our study of the principles of formation of number words we found an extensive use of addition and multiplication, a less use of subtraction, and very slight use of division in the formation of number systems. But aside from these instances (all operations on only the *bases* of the systems) the calculative ability of the American Indians was very slight and of the most elementary sort. Addition, subtraction or multiplication was accomplished only with the aid of the fingers, sticks, pebbles or other convenient counters. It is probable that the native Indian mind had practically no idea of mental arithmetic, being unable to multiply or divide numbers mentally, or even to add or subtract any except the smallest. His need for such operations was probably as slight as his knowledge.

THE CURVE OF LIGHT ON A CORRUGATED DOME.

By WM. H. ROEVER, Washington University.

The Phenomenon. On a bright day a person situated at a high point, within a distance of three or four miles of the new Roman Catholic Cathedral of St. Louis, will observe a well-defined curve of light on the dome of the cathedral. This curve, which is closed, passes through the highest point of the dome and

[1] The Maya of Mexico has at least 75 such classifiers. See article by Mrs. Z. Nuttall in *Amer. Anthropologist*, N.S., Vol. 5, p. 667.

changes its size and position with the changing position of the sun. The accompanying photograph (Fig. 1), taken from a high building about a mile from the Cathedral, shows this closed curve which resembles a halo. The same sort of phenomenon may be observed on the stem of a watch when held in the light.

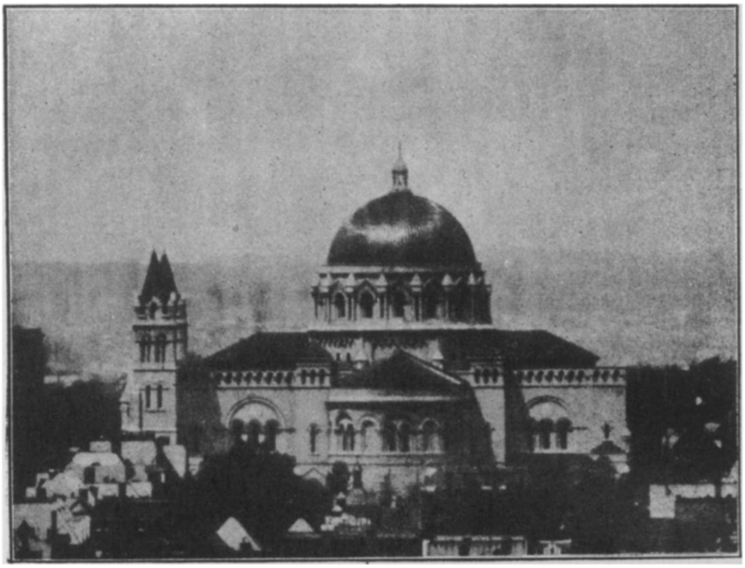

FIG. 1.

The Cause. The dome of the cathedral is covered with glazed tiles laid along those great circles of the spherical dome which pass through the highest point, thus forming a sort of corrugated surface, of which the ridges lie

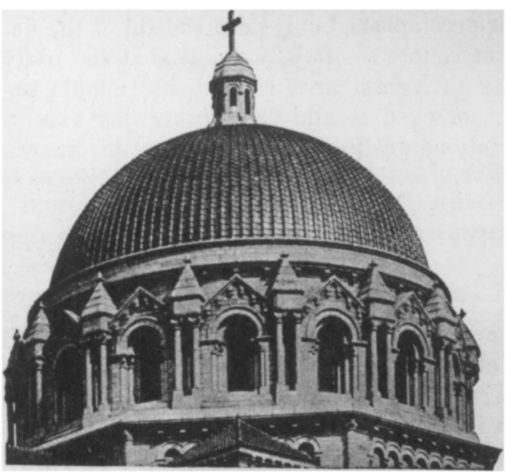

FIG. 2.

along the meridians of the sphere (Fig. 2). The sun's light is reflected from this glazed, corrugated surface, and an observer, properly situated, sees in each corrugation a reflected image of the sun or an actual brilliant point, as this image is sometimes called. The assemblage of these images or brilliant points, any two consecutive ones of which are too near each other to appear separated to the eye of the distant observer, forms the curve or halo described above.

The Mathematical Problem Involved. In order to determine the mathematical nature of this curve of light we must make some assumptions, as we do in the mathematical formulation of every physical problem. It is assumed that the dome is a portion of a spherical surface, in fact, a hemisphere, and that the meridional corrugations are so small and numerous that we may regard the spherical dome as covered with a family of polished wires which are laid along the great circles through the highest point. It is further assumed that these wires are so fine that they may be regarded as mathematical curves. Furthermore, the sun's rays are regarded as a system of parallel rays, and the observer is supposed to be far enough away from the Cathedral so that the reflected rays which reach his eye may also be regarded as belonging to a system of parallel rays. Under these assumptions the mathematical problem involved may be stated as follows:

Determine the locus of the actual brilliant points of the meridians of a sphere with respect to a source of light which is infinitely distant and an observer who is (practically) infinitely distant.

Solution of the Mathematical Problem.

Definition. A point P is said to be an *actual brilliant point* of the curve C (plane or twisted) with respect to the source of light P_1 and an observer P_2 when

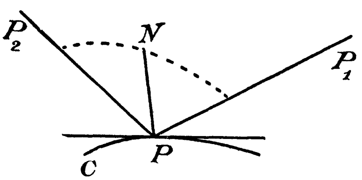

Fig. 3.

the internal bisector of the plane angle P_1PP_2 is a normal to the curve C at the point P. In particular P_1 and P_2 may be infinitely distant.

Accordingly, if we denote by l_1, m_1, n_1 the direction cosines of the directed line PP_1, by l_2, m_2, n_2 the direction cosines of the directed line PP_2, and by l, m, n numbers proportional to the direction cosines of the tangent line to the curve C at the point P, then the condition that the point P should be an actual brilliant point of the curve C, with respect to the source P_1, and the observer P_2, is*

$$(1) \qquad (l_1 + l_2)l + (m_1 + m_2)m + (n_1 + n_2)n = 0.$$

* For more details see the author's paper "Bril ant Points of Curves and Surfaces," *Transactions of the American Mathematical Society*, Vol. 9, No. 2, pp. 245–279.

In our particular problem the source P_1 is the infinitely distant sun, P_2 is the eye of the (practically) infinitely distant observer and the curve C is a vertical great circle, that is a meridian, of the spherical dome. In order to get the form of condition (1) for our problem, let us assume a set of rectangular axes of which the origin lies at the center of the spherical dome and of which the axis of z is vertical and positive in the direction of the zenith. The equations of a meridian then are (Fig. 4)

$$F = 0 \quad \text{and} \quad \Phi = 0, \tag{2}$$

where

$$F = x^2 + y^2 + z^2 - R^2 \quad \text{and} \quad \Phi = \frac{y}{x} - k,$$

in which R denotes the radius of the spherical dome and k is a parameter whose value depends on the particular meridian under consideration.

The direction cosines of a tangent to the curve (2) at a point (x, y, z) of this curve are proportional to the three determinants of the matrix

$$\begin{Vmatrix} \dfrac{\partial F}{\partial x} & \dfrac{\partial F}{\partial y} & \dfrac{\partial F}{\partial z} \\[2ex] \dfrac{\partial \Phi}{\partial x} & \dfrac{\partial \Phi}{\partial y} & \dfrac{\partial \Phi}{\partial z} \end{Vmatrix}.$$

Hence if, as above, we denote by l, m, n numbers proportional to these direction cosines, we have

$$l : m : n = -xz : -yz : x^2 + y^2. \tag{3}$$

Since, for our problem, l_1, l_2, m_1, m_2, n_1, n_2 are constants at any particular instant of time, let us introduce new constants by the relations:[1]

$$\alpha = l_1 + l_2, \quad \beta = m_1 + m_2, \quad \gamma = n_1 + n_2.$$

Hence condition (1) for our problem takes the form:

$$-\alpha xz - \beta yz + \gamma(x^2 + y^2) = 0. \tag{4}$$

This equation represents a quadric cone whose vertex is at the origin of coördinates, and whose horizontal sections are circles. This cone contains as two of its elements the axis of z and the internal bisector of the angle formed at the origin by the rays which pass to the sun and to the observer, and the plane of these two elements is a plane of symmetry. It is the locus of all of the actual brilliant points of the two parameter family of curves obtained by allowing both R and k in equations (2) to be variable parameters. The locus which we are seeking is the intersection of this locus with the sphere:

$$x^2 + y^2 + z^2 - R^2 = 0. \tag{5}$$

Since the dome is the upper hemisphere of the sphere (5), we are interested only

[1] These constants are proportional to the direction cosines of the internal bisector PN of the angle P_1PP_2.

in the upper branch of the curve made by the intersection of (4) and (5). This is the only portion of the locus which we observe. Figure 4 is an orthographic projection in which the sun is represented as being in the xz plane at an altitude of 60° and the observer at an infinite distance in a direction perpendicular to the plane of the drawing.

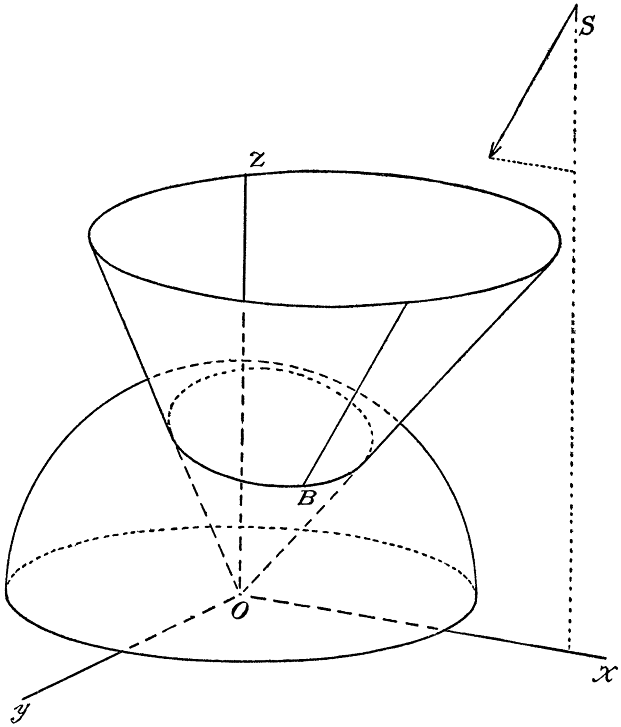

Fig. 4.

The observer is practically in the horizontal plane which passes through the center of the dome. Hence, practically, $n_2 = 0$. When the sun is in the horizon $n_1 = 0$ also, and therefore $\gamma = 0$. Then equation (4) becomes

$$z(\alpha x + \beta y) = 0,$$

and the curve of light (4)(5) degenerates into two great circles of the sphere, of which one is horizontal and the other vertical.

Finally, it is interesting to note that if the corrugations had been parallels instead of meridians, the curve of light would have been a meridian.[1]

Figure 1 shows, in addition to the curve of light on the dome, a line of light on the conical roof of the eastern apse of the Cathedral. This is the element along which a plane perpendicular to the internal bisector of the angle P_1PP_2 (Fig. 3) is tangent to the conical roof.

[1] This will be further elucidated in paper on "Optical Interpretations in Higher Geodesy" soon to be published in the MONTHLY.